Math 141 Practice Final Exam

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Solutions will be provided upon request.

PART (i) : Multiple Choice Questions

Each question has exactly **one** correct answer and there are 10 questions in total.

1. $\displaystyle\int \frac{x^4+1}{x^2+1}dx =$

(A) $\dfrac{x^3}{3} + x + 2\arctan(x) + C$

(B) $\dfrac{x^3}{3} + x - 2\arctan(x) + C$

(C) $\dfrac{x^3}{3} - x - 2\arctan(x) + C$

(D) $\dfrac{x^3}{3} - x + 2\arctan(x) + C$

(E) None of the above

2. The integral $\displaystyle\int_1^\infty \frac{\ln(x)}{x^4}dx$

(A) Diverges

(B) equals $\dfrac{1}{9}$

(C) equals $\dfrac{2}{9}$

(D) equals 1

(E) None of the above

3. Consider the following series:

$$a_n = \sum_{n=1}^{\infty} \frac{1}{(n^3+10)^{\frac{1}{4}}} \quad b_n = \sum_{n=1}^{\infty} \frac{\ln n}{n} \quad c_n = \sum_{n=2}^{\infty} \frac{\cos(\pi n)}{\sqrt{n-1}} \quad d_n = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\frac{3}{2}}}$$

Which of the following statement is **true**?

(A) a_n, b_n, c_n are divergent

(B) c_n, b_n are conditionally convergent

(C) b_n, d_n are absolutely convergent

(D) d_n is conditionally convergent

(E) None of the above

4. Let D be the region bounded by $y = 0, y = \dfrac{4}{x}, x = 1, x = 4$. The volume of the region $\mathcal{D}$ obtained by rotating D around the y axis is

(A) 24π

(B) 32π

(C) 36π

(D) 48π

(E) None of the above

5. What is the value of $\dfrac{d}{dx}\left[\displaystyle\int_1^{x^4} e^t dt\right]$?

(A) $e^{x^4} - e$

(B) e^{x^4}

(C) $4x^3(e^{x^4} - e)$

(D) $4x^3 e^{x^4}$

(E) None of the above

6. $\displaystyle\int \tan^6(x)\sec^4(x)dx =$

(A) $\dfrac{1}{6}\tan^6(x) + \dfrac{1}{8}\tan^9(x) + C$

(B) $\frac{1}{7}\tan^7(x) + \frac{1}{9}\tan^9(x) + C$

(C) $\frac{1}{7}\sec^7(x) + \frac{1}{9}\sec^9(x) + C$

(D) $\frac{1}{6}\sec^6(x) + \frac{1}{8}\sec^9(x) + C$

(E) None of the above

7. The equation of an ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a \neq b$ and $a, b \neq 0$. What is the area bounded by this ellipse?

(A) $\pi(a + b)$

(B) $\frac{\pi}{4}ab$

(C) πab

(D) $\pi\sqrt{ab}$

(E) None of the above

8. The series $\sum_{n=1}^{\infty}(-1)^n\frac{(2n)!}{(n!)^2}$ is

(A) Divergent

(B) Absolutely Convergent

(C) Conditionally Convergent

(D) Decreasing for all $n \geq 1$

(E) None of the above

9. The interval of convergence of the series given by $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{\sqrt{n}2^n}$ is

(A) $\left(-\frac{1}{2}, \frac{1}{2}\right]$

(B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $(-2, 2]$

(D) $[-2, 2]$

(E) None of the above

10. Figure 1 shows a semicircle with radius 1, a horizontal diameter PQ and two tangent lines at P, Q. A horizontal line is placed in between two tangent lines, resulting in shaded area (in blue). At what height above PQ should this horizontal line be such that the shaded area is minimized?

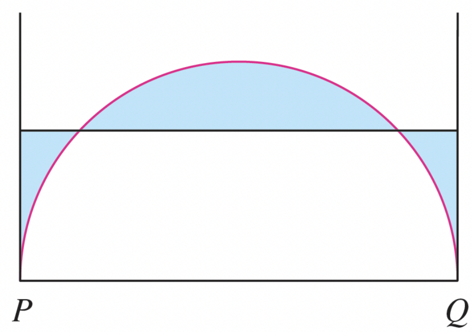

Figure 1: Figure for Question 10.

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{\sqrt{3}}{3}$

(E) None of the above

PART (ii) : Long Answer Questions

There will be 7 questions in total.

11. Using the standard definition of Riemann sums, calculate $\int_0^1 (2x+1)dx$.

12. Find the arc length of the curve $y = \ln(\cos x)$ where $0 \leq x \leq \dfrac{\pi}{3}$

13. Evaluate the following integrals:

(**i**) $\displaystyle\int x \sin x \cos x dx$ (**ii**) $\displaystyle\int e^x \cos(x) dx$ (**iii**) $\displaystyle\int \frac{1-\tan(x)}{1+\tan(x)} dx$

14. Determine whether the following series are divergent, conditionally convergent or absolutely convergent:

(**i**) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+\sqrt{n+\sqrt{n}}}}{\left(n+(n+n^2)^2\right)^2}$ (**ii**) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\cos(n\pi)}{n}$ (**iii**) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{\ln n}}$

15. Find the surface area of the solid of revolution obtained by rotating the curve $y = \sqrt{1+e^x}$ where $0 \leq x \leq 1$ about the x-axis.

16. An infinite series is given by $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!}$ where $k > 0$. Find all values of k such that the series is absolutely convergent.

17. Recall the property of geometric series: Let $a_n = x^n$, then

$$\sum_{n=0}^{\infty} a_n = \frac{1}{1-x}, \quad \text{when } x \in (0,1).$$

Use this property to show that, when x is close to 0^+, we have $\arctan(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$.